



Techno Science Africana Journal

Journal homepage: www.technoscienceafricana.com



MODEL DEVELOPMENT AND OPTIMIZATION OF REFINERY PRODUCTION SCHEDULING

Aminu A. Hamisu¹, A. Mohammad², S. Bilal³, U. Abubakar⁴ and Yusuf A. Sha'aban⁵

^{1,3,4}Department of Chemical Engineering, Ahmadu Bello University, Zaria, Nigeria

^{2,5}Department of Electrical and Computer Engineering, Ahmadu Bello University, Zaria, Nigeria

ARTICLE INFO

ABSTRACT

Keywords:

Refinery scheduling, intermediate product blending, MINLP formulation; discrete time representation; global optimization.

In refinery scheduling, model development is in most cases tied to the solution procedure devised to solve the model. Most of the existing methodologies are based on: a simple approach that decompose a large problem into smaller size subproblems and treat the subproblems separately; a sequential approach that allow exchange of information between subproblems and integrate the two; and a more rigorous procedure that solves two subproblems simultaneously. To perform operations in the most efficient way, simultaneous optimization of production with intermediate product blending is necessary. This is to ensure reliable schedules are generated while satisfying physical and economic constraints. In this paper, a novel scheduling model for refinery production with intermediate products blending was developed as MINLP and solved with the view to obtaining optimal allocation of materials, distribution of resources, assignments of tasks and processing times of different crude slates. A large scale refinery problem of 10 days scheduling horizon length was used as a case study to test the performance of the model. The scheduling horizon is discretized into a number of time periods of variable length and then equal length of 1 day was considered thereafter for comparison. The model was solved using Sobolopt global optimizer (GAMS-based solver). The results obtained have shown that reliable decisions can be obtained for implementation in real plants.

1. INTRODUCTION

Optimal scheduling of refinery production processes is of paramount importance in order to maximize the overall refinery profit. Developing an optimization model is necessary to achieve the objective of obtaining the maximum profit. Generally, refinery production scheduling models are formulated as nonlinear programming problems with time representation as discrete or continuous and the solution is obtained using simple, sequential or integrated approaches. In addition to being nonlinear, refinery scheduling models involve discrete

and continuous decisions; hence formulated as mixed integer nonlinear programming (MINLP) problems. Models presented in Pinto *et al.* (2000); Méndez *et al.* (2006); Li *et al.* (2010); and Cuiwen *et al.* (2013) are all MINLPs. The main difference in the aforementioned refinery production models lie in the time formulation adopted (discrete or continuous), problem size and complexity (large-scale or small-scale and the number of units involved), and the solution strategy devised to ensure feasibility and optimality.

Discrete time formulation requires that the length of the scheduling horizon be divided into a number of time intervals of equal or varying duration. In this formulation, the beginning and ending of all activities or events are forced to coincide with the boundaries of the time intervals.

Binary variables are used to model the occurrence or non-occurrence of events at these time interval boundaries. In some way, discrete time formulation simplifies the model but introduces more binary variables making larger problems intractable or difficult to solve. The refinery scheduler has to decide on the length of the time interval to use based on the nature of the problem at hand. Larger time intervals oversimplify the problem giving inaccurate results. The use of smaller time intervals gives a more reasonable solution but requires higher computational efforts as reported in Hamisu *et al.* (2013).

Unlike the discrete time formulation, the continuous time representation allows events to occur at any time within the time interval boundaries. The idea is to split the time horizon into several variable time slots so that events are allowed to occur at the boundaries of the time intervals. Such formulations introduce flexibility and possibly eliminate unnecessary event time interval assignments, thus resulting in problem much smaller in size compared with its equivalent discrete time representation and hence require less computational efforts(Floudas and Lin, 2004). Despite these benefits however, variable event timing makes continuous time models difficult to handle as inventory and material balance have to be checked at each time slot to ensure solution feasibility(Méndez *et al.*, 2006). Considering the advantages and limitations in the use of the aforementioned time representations, the choice between the two is still an open debate.

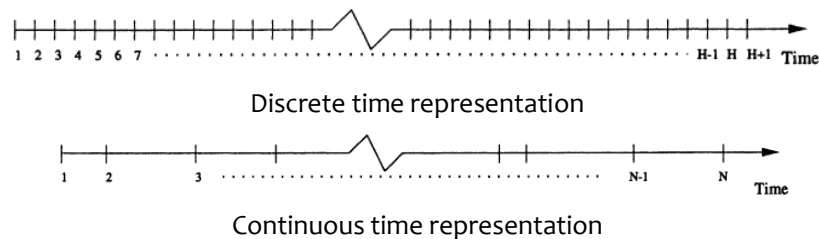


Figure 1: Discrete and Continuous time representations(Floudas and Lin, 2004)

Irrespective of the time representation adopted in model development, the number of equations and variables in the model depends on the size and complexity of the problem the refinery scheduler is attempting to solve. In research studies, a number of approaches are employed to handle large-scale problems. They are:

- A simple approach that decompose a large problem into smaller size subproblems and treat the subproblems separately.
- A sequential approach that allow exchange of information between subproblems and integrate the two and,
- A more rigorous procedure that solves two subproblems simultaneously.

Of the approaches mentioned above, the third one is mostly preferred as it integrates all subsystems in a plant. In refinery, integrating

production with intermediate products blending will allow more realistic exchange of information and flow of material between two interconnecting units. This generates more reliable schedules and offers comprehensive cost minimization options that allow production to follow correct operational sequence.

Despite the benefit inherent in simultaneous optimization of production with blending, it appears fewer publications have been reported in this area of research. Planning/scheduling models for production and products blending subsystems have been developed by Aronofsky *et al.* (1978); Kendrick *et al.* (1981); Moro *et al.* (1998); Pinto *et al.* (2000); Joly *et al.* (2002); Li *et al.* (2005); Alattas *et al.* (2012) and Tong *et al.* (2012). These methodologies depict the whole plant

topology but the models differ in the way the optimization problem is formulated, size of the problem, complexity of the units involved and the objectives aimed to achieve. Some scheduling models of production with blending subsystems focused on specific product orders and were developed based on a specific time formulation. Also, other models for scheduling are formulated using fewer numbers of units.

Scheduling model presented by Jia and Ierapetritou (2004) for refinery production subsystem considers fractionation and reaction processes using continuous time formulation. The major drawback of their approach is the non-inclusion of other vital units like FCC, NHU and CRU; making the model unreliable and hence inaccurate. A simultaneous slot based short-term scheduling and off-line blending of gasoline products model has been reported in Méndez *et al.* (2006) in both discrete and continuous time domain in which an integrated MINLP problem is solved as iterative MILPs.

Another MINLP slot based formulation that relies on iterative MILP solution procedure is the work of Li *et al.* (2010). Also, their model is based on gasoline product but incorporate many real life features like changeovers, non-identical blenders, minimum run length etc. Cuiwen *et al.* (2013) developed mathematical formulations as MINLP and devises a solution algorithm that solves real world refinery scheduling problems for production with diesel blending. It is important to note that in real life refinery operation, other products like gasolines are also blended in addition to the diesel product grades. Hence, the major setback in their approach is that the model was developed based on refinery configuration that processes only diesel products. However, the MINLP model was able to capture both certain and uncertain events.

Considering the gap of knowledge highlighted above, this paper is aimed at developing a multi-product refinery scheduling model that captures all the necessary interactions between production units and blending subsystem while solving the two simultaneously using a global optimizer.

2. PROBLEM DEFINITION AND MODEL FORMULATION

In this section, a novel MINLP scheduling model for refinery production with intermediate products blending is developed to optimize the allocation and distribution of resources, assignments of tasks and processing times of different crude oil. The CDU model is based on fixed yield approach and the objective of the optimization model is to maximize profit while generating feasible schedules within a given time horizon.

2.1. Problem Definition

The following technical details are assumed to be available:

1. A scheduling horizon $[0, H]$.
2. Crude oil compositions and limits on their flow rates.
3. Stream connections.
4. Costs of raw materials: crude oil feeds and butane additives.
5. Unit capacities of CDU and other downstream units along with their operational costs.
6. Yield coefficients of all streams generated from the main refinery production units.
7. Bounds on the intermediate streams properties.
8. B blenders with specification on the component streams to be received at any given time period.
9. P products with specification on blender processing each of these products. Lower limits on the blend times and production rates have to be specified as well. Each blender is assigned to feed a specific product tank and receives input from specific components streams.
10. Specification limits on the products property indices and revenue realised from the products sales.
11. Q Product tanks with specification on blenders each storage tank should receive from. Limits on volume of product in each storage tank at any given time including time zero and demand for various products are specified.
12. Product tanks inventory costs.
13. S sales, their constituent products, amounts, and delivery time windows.

With these technical details, the problem here is to find feasible set of operations that maximize the overall refinery profit by determining the following decision variables:

1. The type and amount of crude oil processed over time.
2. Components flowrates and their quality indices.
3. Flow profiles of feeds into blending units.
4. Production volume of each blender over time, and the blending duration.
5. The inventory profiles of product tanks.
6. The amount of products available for sale during the scheduling horizon.

2.2. Mathematical Model Development

The motivation here is to come up with a refinery scheduling model that captures all the necessary interactions between production units and blending subsystem while solving the two subsystems simultaneously.

The aforementioned objectives cannot be achieved without some assumptions and consideration for certain operating rules. They are:

2.2.1. Operating Rules

1. CDU and other upstream units are in continuous operation during the scheduling horizon.
2. Blenders operate in a semi-continuous mode. When running, blender is connected to product storage tank and no sales of product at that period of time. When idle, blender is connected to a dummy storage tank and product stored in real tank will be available for sale.
3. A blender cannot process more than one product at any time.

4. A blender cannot feed more than one product tank at any time.
5. Products from different blenders can be processed at same time. Such decision is vital to avoid using tanks unnecessarily.
6. A blender cannot be fed while discharging products.
7. While feeding a blender, a product cannot be discharged at same time.

2.2.2. Model Assumptions

1. The flow of components streams is such that it meets minimum amount required for blending operation for the whole time periods.
2. Perfect mixing assumption for all blenders.
3. No changeover operations for blenders since specific component streams are assigned to each blender.
4. No changeover for product storage tanks since each tank is allocated to a specific product.
5. Each product is processed within the scheduling horizon.
6. All sales involve only one product at a time and multiple products sales can be decomposed into several single-product sales.

2.2.3. Mass Balance for Crude Oil

The amount of crude oil to be supplied must satisfy its minimum $FB_{cr,t,min}$ and maximum $FB_{cr,t,max}$ processing requirements. This however, depends also on the crude oil processing time $\varphi_{cr,t}$ and composition $\omega_{cr,t}$. These are represented by the following constraints:

$$FB_{cr,t} \geq FB_{cr,t,min} \varphi_{cr,t} \omega_{cr,t} \quad \forall cr, t \tag{1}$$

$$FB_{cr,t} \leq FB_{cr,t,max} \varphi_{cr,t} \omega_{cr,t} \quad \forall cr, t \tag{2}$$

While $FB_{cr,t,min}$ and $FB_{cr,t,max}$ are in bbl per day, the crude oil processing time $\varphi_{cr,t}$ is a fraction of a day. Processing time of crude oil depends on the binary variable $XC_{cr,t}$ which decides whether crude cr is processed at time period t and the length of the production period LT_t .

$$\varphi_{cr,t} = LT_t XC_{cr,t} \quad \forall cr, t \tag{3}$$

The sum of the production period lengths over all time periods gives the length of the scheduling horizon H .

$$H = \sum_t LT_t \tag{4}$$

2.2.4. Downstream Units

The yield, capacity, and quality constraints of other refinery units are formulated in accordance with the stream connections in the refinery plants:

The yield of any intermediate product $F_{s',u,cr,t}$, which is the output stream s' from unit u is obtained by multiplying the yield coefficient $\eta_{s',u,cr,t}$ with input stream $F_{s,u,cr,t}$.

$$F_{s',u,cr,t} = \eta_{s',u,cr,t} F_{s,u,cr,t} \quad \forall u, cr, t \quad (5)$$

A stream feeding a unit must satisfy the unit capacity cap_u requirement

$$\sum_{s'} F_{s',u,cr,t} \leq cap_u \quad \forall u, cr, t \quad (6)$$

Streams are connected from one unit to another by means of splitters and mixers.

For splitters the following constraint holds:

$$F_{s,split,cr,t} = \sum_s F_{s',split,cr,t} \quad \forall split, cr, t \quad (7)$$

Similarly, for mixers the constraint is represented as

$$\sum_s F_{s',mix,cr,t} = F_{s,mix,cr,t} \quad \forall mix, cr, t \quad (8)$$

A special mixer is the blending unit (blender) that processes component streams from splitters into blends of different quality attributes kas specified by customers. Output streams from blenders $F_{p,cr,t}$ are the refinery main products available for sale.

$$\sum_{s,u} F_{s',u,cr,t} = F_{p,cr,t} \quad \forall p, cr, t \quad (9)$$

Product properties at time t , $\zeta_{k,p,t}$ must meet minimum or maximum specifications and are related to the component streams and additives properties $\xi_{k,s,u,cr,t}$.

Properties like vapour pressure, density, and sulphur in product streams must be less than or equal to a certain value. Octane rating on the other hand must be greater than or equal to some specific number.

$$\zeta_{k,p,t} F_{p,cr,t} \leq \sum_{s,u} \xi_{k,s,u,cr,t} F_{s,u,cr,t} \quad \forall p, cr, t \quad (10)$$

$$\zeta_{k,p,t} F_{p,cr,t} \geq \sum_{s,u} \xi_{k,s,u,cr,t} F_{s,u,cr,t} \quad \forall p, cr, t \quad (11)$$

2.2.5. Blending Operation

A blender operates in a semi-continuous mode and therefore at any time instant, the blender is either running or idle. When running, blender is connected to a product tank. Binary variables are defined to model blender when in operation and when in idle mode.

$$x_{b,p,t} = \begin{cases} 1 & \text{if blender } b \text{ is processing product } p \\ & \text{at time period } t \\ 0 & \text{otherwise} \end{cases} \quad \forall b, p, t \quad (12)$$

$$y_{b,q,t} = \begin{cases} 1 & \text{if blender } b \text{ is feeding product tank } q \\ & \text{at time period } t \\ 0 & \text{otherwise} \end{cases} \quad \forall b, q, t \quad (13)$$

$$z_{q,p,t} = \begin{cases} 1 & \text{if product tank } q \text{ is storing product } p \\ & \text{at time period } t \\ 0 & \text{otherwise} \end{cases} \quad \forall q, p, t \quad (14)$$

The following relations hold true:

At any time period, if blender b feeds tank q and that tank stores product p then b processes p at that particular time period. i.e.

$$x_{b,p,t} \geq z_{q,p,t} + y_{b,q,t} - 1 \quad \forall b, p, q, t \quad (15)$$

If b processes p and feeds q at time period t , then q holds p at that particular time period.

$$z_{q,p,t} \geq x_{b,p,t} + y_{b,q,t} - 1 \quad \forall b, p, q, t \quad (16)$$

Each blender b cannot feed more than one product tank q at time period t . i.e.

$$\sum_q y_{b,q,t} \leq 1 \quad \forall b, t \quad (17)$$

Blender cannot receive component streams while feeding storage tank q . A binary variable $cs_{s,u,cr,t}$ is defined to denote that blender b is receiving component streams s of crude cr from refinery production unit u at time period t .

$$cs_{s,u,cr,t} = \begin{cases} 1 & \text{if blender } b \text{ is receiving from component} \\ & \text{streams at time period } t \\ 0 & \text{otherwise} \end{cases} \quad \forall b, cr, t \quad (18)$$

The following constraint ensures that blender is either receiving component streams or feeding product tank.

$$cs_{s,u,cr,t} + y_{b,q,t} \leq 1 \quad \forall b, cr, q, t \quad (19)$$

Due to semi-continuous nature of its operation, not all the component streams are processed at the same time in the blender. Component streams that cannot be processed at a particular time period are temporarily stored to be used at a later time. Therefore each component stream splits into two non-negative variables. The amount processed by the blender at any time period is the fraction of the component streams received at that particular time. Equation 5-14 can then be transformed to:

$$\sum_{s,u} F_{s,u,cr,t} = F_{blend,cr,t} cs_{s,u,cr,t} + F_{stored,cr,t} \quad \forall b, cr, t \quad (20)$$

The actual amount produced during blending operation $FP_{p,t}$ must satisfy the following constraint

$$FP_{p,t} x_{b,p,t} \leq F_{blend,cr,t} \quad \forall b, p, t \quad (21)$$

Therefore, Equations (10) and (11) can be re-written as

$$\zeta_{k,p,t} FP_{p,t} \leq \sum_{s,u} \xi_{k,s,u,cr,t} F_{blend,cr,t} \quad \forall k, p, t \quad (22)$$

$$\zeta_{k,p,t} FP_{p,t} \geq \sum_{s,u} \xi_{k,s,u,cr,t} F_{blend,cr,t} \quad \forall k, p, t \quad (23)$$

During operation, a blender runs within its lower $RL_{p,t,min}$ and upper limits $RL_{p,t,max}$ and is related to the product processing time $\phi_{p,t}$.

$$\phi_{p,t} \geq RL_{p,t,min} \quad (24)$$

$$\phi_{p,t} \leq RL_{p,t,max} \quad (25)$$

The volume $VP_{p,t}$ produced from each blender depends on the amount processed by the blender (in bbl per day) and the product processing time (in hours). Thus,

$$VP_{p,t} = FP_{p,t} \phi_{p,t} x_{b,p,t} \quad (26)$$

The volume produced must be within its production limit and should not be more the capacity of the blender $cap_{blender}$.

$$VP_{p,t} \geq VP_{p,t,min} \quad (27)$$

$$VP_{p,t} \leq VP_{p,t,max} \quad (28)$$

$$VP_{p,t} \leq cap_{blender} \quad (29)$$

2.2.6. Product Storage and Inventory

From mass balance, a material inventory of each product in storage tank $INPT_{p,t}$ can be accounted for. This is calculated as the sum of the initial inventory in the tank at the beginning of each time period and the actual amount produced less the actual amount sold to customers at the same period.

$$INPT_{p,t+1} = INPT_{p,t} + VP_{p,t} - VPS_{p,t} \tag{30}$$

$VPS_{p,t}$ is the actual amount of each product available for sale to customers at time period t . This amount must meet the demand $DM_{p,t}$ of customers throughout the scheduling horizon.

$$VPS_{p,t} \geq DM_{p,t} \tag{31}$$

The inventory must always be within products' lower and upper limits and must not exceed the tank capacity VPT_p .

$$INPT_{p,t} \geq INPT_{p,t,min} \tag{32}$$

$$INPT_{p,t} \leq INPT_{p,t,max} \tag{33}$$

$$INPT_{p,t} \leq VPT_p z_{q,p,t} \tag{34}$$

As noted in Hamisu et al. (2013), it is an operational policy to keep minimum safety stock above tanks bottom levels in order to safeguard floating device. To prevent tanks from running out of stock at the end of the scheduling horizon, periodic constraint on the inventory is imposed. i.e.

$$INPT_{p,t+1} \leq INPT_{p,1} \tag{35}$$

This ensures that low inventory at the end of the scheduling horizon is avoided. Rebennack et al. (2011) recommends that only one of the storage tanks safety stock values is fixed while leaving the values for other tanks as variables to be determined by model solution. This is to avoid being trapped in the infeasibility region.

2.2.7. Objective Function

The objective function is computed as the overall refinery profit PR which is the revenue generated from selling the products minus all the costs associated with raw material, unit operation, inventory, purchased intermediate (additive), and penalties.

$$PR = \sum_t \sum_p pf_p * VPS_{p,t} - \sum_t \sum_{cr} ct_{cr} * FB_{cr,t} * \varphi_{cr,t} - \sum_t \sum_u g_u \tag{36}$$

$$* F_{s,u,cr,t} * \varphi_{cr,t} - \sum_t \sum_p pst_p * INPT_{p,t} - \sum_t ui * BUT_t$$

$$- \sum_t penalties$$

The revenue is computed as product of price of product per bbl pf_p and the total amount sold to customers $VPS_{p,t}$. Raw material cost is obtained by multiplying the cost per bbl ct_{cr} with the amount of crude oil processed $FB_{cr,t}$ and the processing time $\varphi_{cr,t}$. Similarly, unit operation cost is the product of cost per bbl g_u processed in each unit, amount processed in each unit $F_{s,u,cr,t}$ and the processing time $\varphi_{cr,t}$. Inventory cost is computed by multiplying inventory cost per bbl stored pst_p with the amount stored $INPT_{p,t}$. Purchased intermediate (butane) cost is obtained in a similar way, multiplying butane cost per bbl ui

and the amount used BUT_t . Penalty is incurred when quality requirements of the products are not met.

3. CASE STUDY

A case study of a real refinery that processes two crude oil grades (light and heavy) is considered here.

The refinery plant integrates production area with blending subsystem. Figure 2 is the process schematic of the material flow in the oil refinery plant. In the figure, crude oil is separated in CDU into straight-run (SR) fuel gas, SR naphtha, SR gasoline, SR distillate, SR gas oil and SR residuum.

Of the crude oil types, Crude 1 is more expensive with low sulphur content (sweet crude) compared with Crude 2 with high sulphur content (sour crude). The scheduling

horizon is of 10 days. The limits on the processing unit capacities are given in Table 1. Other technical details for use in the model are given in Tables (2) to (9).

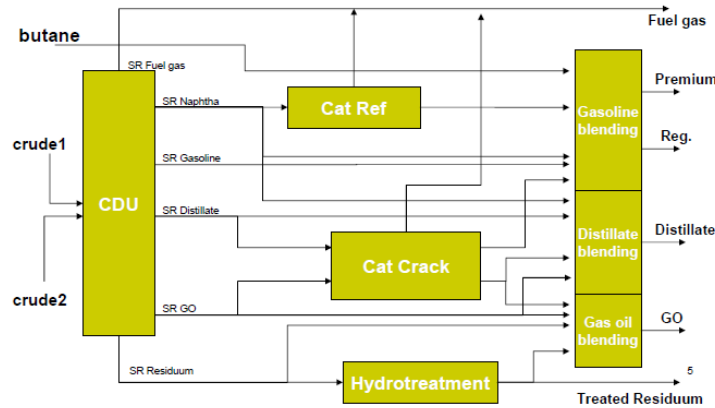


Figure 2: Refinery production with product blending (Alattas et al., 2011)

The lighter products such as SR gasoline and SR fuel gas are sent directly to the blending unit or markets. The intermediate products such as SR distillate and SR gas oil are catalytically cracked in FCC and then sent to the blending unit. SR naphtha is catalytically reformed in CRU, and then sent to the blending unit. Part of the SR residuum is sent to the hydro-treater unit for removal of sulphur. The processed residuum is then fed to the blending unit. The final products from these processes are fuel gas, premium gasoline, regular gasoline, distillate, and gas oil. With the exception of fuel gas, these final products are blends which meet quality specifications such as octane level, vapour pressure, density and sulphur content. Also in Figure 2, there is a butane stream, a material with high octane rating for use in gasoline blending.

Two (2) different blending and production scheduling problems are formulated and presented in this section in order to evaluate the efficiency of the proposed model. The first case presents problem with time periods of varying duration in the scheduling horizon. In this formulation, the time horizon splits into several varying time slots so that events are allowed to occur at the boundaries of the time intervals, i.e. events do occur at any time within the time interval boundaries. The second case involves a problem formulation using discrete time representation. The discrete time formulation requires that the length of the scheduling horizon is divided into a number of time intervals of equal duration. The cases are to be solved based on global optimization approach.

Table 1: Capacities of process units (1000 bbl/day)

Crude distillation	100
Catalytic reformer	20
Catalytic cracker	30
Product tank	150

Table 2: Crude oil availability (1000s bbl/day)

Limit/days	1	2	3	4	5	6	7	8	9	10
Minimum	20	20	20	20	20	20	20	20	20	20
Maximum	200	200	200	200	200	200	200	200	200	200

Table 3: Refinery raw material/operating costs and product prices

Raw material costs (\$/bbl)	
Crude 1	75
Crude 2	65
Butane	67.5
Operating costs (\$/bbl processed)	
Crude distillation	5
Catalytic reformer	7.5
Catalytic cracker	40
Catalytic cracker - Gas oil feed	4
Hydrotreater - Distillate feed	5
Inventory cost	0.00306
Product prices (\$/bbl)	
Premium gasoline	135
Regular gasoline	121
Distillate	87
Gas oil	76.5
Fuel gas	35

Table 4: Yield patterns for the downstream process units

Output yields	Crude 1	Crude 2
Catalytic reformer		
Reformed fuel gas	0.129	0.099
Reformate for gasoline blending	0.807	0.836
Loss	0.064	0.065
	1.000	1.000
Catalytic cracker		
	DSGO	DS GO
Cracked fuel gas	0.30 0.31	0.36 0.38
Cracked gasoline	0.59 0.59	0.58 0.60
Gas oil (for distillate or gas oil blending)	0.21 0.22	0.15 0.15
	1.100 1.12	1.09 1.09
Hydrotreater		
Desulphurized residuum		RS 0.97

Table 5: Blending information from yields

Premium and Regular Gasoline	Clear Research Octane		Vapour Pressure (mmHg)	
SR gasoline (from crude unit)	78.5		18.4	
SR naphtha (from crude unit)	65.0		6.54	
Reformate (from reformer)	104.0		2.57	
Cracked gasoline (from cat cracker)	93.7		6.90	
Butane	91.8		199.2	
	Density (lb/bbl)		Sulphur (lb/bbl)	
<i>Distillate</i>	Crude 1	Crude 2	Crude 1	Crude 2
SR naphtha (from crude unit)	272.0	272.0	0.283	1.48
SR distillate (from crude unit)	292.0	297.6	0.526	2.83
SR gas oil (from crude unit)	295.0	303.3	0.980	5.05
Cracked gas oil	294.4	299.1	0.353	1.31
<i>Gas oil blend</i>				
Cracked gas oil	294.4	299.1	0.353	1.31
SR gas oil	295.0	303.3	0.980	5.05
SR residuum	343.0	365.0	4.700	11.00
Hydrotreated residuum		365.0		6.00

Table 6:Product quality specifications

	Clear Research Octane	Vapour Pressure (mmHg)	Density (lb/bbl)	Sulphur (lb/bbl)
Premium gasoline	(≥) 90	(≤) 12.7		
Regular gasoline	(≥) 86	(≤) 12.7		
Distillate			(≤) 306	(≤) 0.5
Gas oil			(≤) 352	(≤)3.5

Table 7:Minimum run length for each blending operation (per day)

Products/days	1	2	3	4	5	6	7	8	9	10
Premium gasoline	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
Regular gasoline	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
Distillate	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
Gas oil	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
Fuel gas	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 8:Demand of final products (1000s bbl per day)

Products/days	1	2	3	4	5	6	7	8	9	10
Premium gasoline	42.3	42.3	42.3	42.3	42.3	42.3	42.3	42.3	42.3	42.3
Regular gasoline	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
Distillate	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
Gas oil	36.8	36.8	36.8	36.8	36.8	36.8	36.8	36.8	36.8	36.8
Fuel gas	17.5	17.5	17.5	17.5	17.5	17.5	17.5	17.5	17.5	17.5

Table 9: Bounds on amount produced and tank inventory (1000s)

	Volume flow of products (bbl/day)		Inventory of products (bbl)	
	Minimum	Maximum	Lower	Upper
Premium gasoline	10	80	30	80
Regular gasoline	10	80	25	60
Distillate	10	80	10	60
Gas oil	10	80	10	60
Fuel gas	10	80	15	80

3.1. Formulation with Variable Time Slots

A large-scale refinery scheduling problem based on variable time formulation is presented in this section to test the performance of the model developed in this paper. The model equations along with the technical details for the large-scale refinery problems were implemented in GAMS v 23.9.1 on a 4GB RAM dual core i5 processor computer. The equations are then solved using Sobolopt global optimizer. The optimal

operating profit of processing 1,883,090 bbl of crude oil in CDU for 10 days was obtained in 4.680 seconds as \$37,920,577. The different components that account for the revenue and expenses are presented in Table 10.

The model generates a profitable production schedule. The information on the optimal crude oil processing rates/ processing times, product sales, and product inventories are summarized in Tables 11, 12 and 13 respectively.

Table 10: Income and costs generated for the whole scheduling horizon (\$1000)

Parameter	Amount
Profit	37,920.577
Sales	282,599.74
Raw material cost	132,894.934
Unit operation cost	19,730.163
Inventory cost	3.629
Purchased intermediate	913.603
Other cost (penalties)	91,136.834

Table 11: Raw material processing information

Period	Length (day)	Crude 1 Processed			Crude 2 Processed		
		Rate (1000s bbl/day)	Volume (1000s bbl)	Time (hr)	Rate (1000s bbl/day)	Volume (1000s bbl)	Time (hr)
1	0.909	181.818	165.273	21.818	18.182	16.527	21.818
2	0.909	181.818	165.273	21.818	18.182	16.527	21.818
3	1.000	200.000	200.000	24.000	0.000	0.000	0.000
4	0.909	181.818	165.273	21.818	18.182	16.527	21.818
5	0.909	181.818	165.273	21.818	18.182	16.527	21.818
6	1.000	0.000	0.000	0.000	200.000	200.000	24.000
7	1.000	0.000	0.000	0.000	200.000	200.000	24.000
8	1.000	200.000	200.000	24.000	0.000	0.000	0.000
9	0.953	190.656	181.695168	22.879	0.000	0.000	0.000
10	1.410	64.227	90.56007	33.849	28.207	39.772	33.849

Table 12: Product sales

Period	Product sales (1000s bbl)				
	Premium gasoline	Regular gasoline	Distillate	Gas oil	Fuel gas
1	42.3	10	60	52.769	145.000
2	42.3	10	10	36.8	80
3	42.3	10	10	36.8	80
4	42.3	10	10	36.8	80
5	42.3	10	10	36.8	80
6	42.3	10	10	65.522	80
7	42.3	10	10	73.505	80
8	42.3	10	10	80	80
9	42.3	10	10	69.307	80
10	42.3	35	20	90	95

Table 13: Product inventory levels

Period	Product inventories (1000s bbl)				
	Premium gasoline	Regular gasoline	Distillate	Gas oil	Fuel gas
1	30	25	60	60	80
2	32.183	25	10	37.253	15
3	34.366	25	10	30.475	15
4	37.164	25	10	26.699	15
5	39.34	25	10	19.319	15
6	43.785	25	10	10	15
7	36.65	25	10	10	15
8	30	25	10	10	15
9	32.185	25	10	10	15
10	32.173	25	10	10	15

3.2. Consideration for Discrete-time with Equal Interval Length

The result presentations and discussions in the preceding section were based on variable time formulation of the scheduling problem. Results for an equivalent formulation with equal interval length of time periods are presented here. An equal interval length of 1 day for the whole scheduling horizon was considered. That is, the 10 day scheduling horizon is discretized into 10 time periods of equal length. The scheduling problem is then

solved again and an optimal profit of \$29,434,980 is obtained. This is less than the amount \$ 37,920,577 computed for the variable length time formulation. The interest here is not to compare the two solutions directly since the difference in the time formulation means the two problems are not exactly the same, but to show the benefit of using a better approach. Optimal crude oil processing details for this uniformly discretized problem are given in Table 14.

Table 14: Raw material processing information

Period	Length	Crude 1 processed			Crude 2 processed		
		Rate (1000s bbl/day)	Volume (1000s bbl)	Time (hr)	Rate (1000s bbl/day)	Volume (1000s bbl)	Time (hr)
1	1	180.000	180.000	24.000	20.000	20.000	24.000
2	1	180.000	180.000	24.000	20.000	20.000	24.000
3	1	0.000	0.000	0.000	200.000	200.000	24.000
4	1	0.000	0.000	0.000	200.000	200.000	24.000
5	1	180.000	180.000	24.000	20.000	20.000	24.000
6	1	0.000	0.000	0.000	200.000	200.000	24.000
7	1	0.000	0.000	0.000	200.000	200.000	24.000
8	1	200.000	200.000	24.000	0.000	0.000	0.000
9	1	0.000	0.000	0.000	196.081	196.081	24.000
10	1	78.480	78.480	24.000	20.000	20.000	24.000

Comparing the two time formulations of the scheduling problem in terms of cost, the revenue generated with variable timing is higher than the amount obtained using uniform time discretization. From Table 11, it

can be observed from the second column that variable time formulation records the actual duration of each task in a given time period and therefore reflects the actual processing times and hence the amount processed at each time period. This is not the case for

uniform time discretization as events are forced to take place at the boundary of intervals. Despite the benefits of variable time formulation as demonstrated in this particular problem, the approach may be inappropriate in other problems.

4. CONCLUSION

This paper reported a novel MINLP scheduling model for production area with blending unit to optimize the allocation of materials, distribution of resources, assignments of tasks

5. REFERENCES

- Alattas, A.M., Grossmann, I.E. & Palou-Rivera, I., (2011). Integration of Nonlinear Crude Distillation Unit Models in Refinery Planning Optimization. *Industrial & Engineering Chemistry Research*, 50(11), pp.6860–6870.
- Alattas, A. M., Grossmann, I. E. & Palou-Rivera, I., (2012). Refinery Production Planning: Multiperiod MINLP with Nonlinear CDU Model. *Industrial & Engineering Chemistry Research*, 51(39), pp.12852–12861.
- Aronofsky, J. S., Dutton, J. M. & Tayyabkhan, M.T., (1978). *Managerial Planning with Linear Programming*, New York: John Wiley and Sons.
- Cuiwen, C., Xingsheng, G. & Zhong, X., (2013). A Data-Driven Rolling-Horizon Online Scheduling Model for Diesel Production of a Real-World Refinery. *AIChE Journal*, 59(4), pp.1160–1174.
- Floudas, C. A. & Lin, X., (2004). Continuous-time versus discrete-time approaches for scheduling of chemical processes: a review. *Computers & Chemical Engineering*, 28(11), pp.2109–2129.
- Hamisu, A. A., Kabantiok, S. & Wang, M., (2013). Refinery scheduling of crude oil unloading with tank inventory management. *Computers & Chemical Engineering*, 55, pp.134–147.
- Jia, Z. & Ierapetritou, M., (2004). Efficient short-term scheduling of refinery operations based on a continuous time formulation. *Computers & Chemical Engineering*, 28(6-7), pp.1001–1019.
- and processing times of different crude slates in a petroleum refinery. The model was formulated based on varying and equally discretised time representations. The performance of the model was tested by solving a large scale refinery problem for scheduling horizon of 10 days using Sobolopt global optimizer (GAMS-based solver). The results obtained have shown that variable time formulation of the problem gives a better solution.
- Joly, M., Moro, L. & Pinto, J., (2002). Planning and scheduling for petroleum refineries using mathematical programming. *Brazilian Journal of Chemical Engineering*, 19(02), pp.207–228.
- Kendrick, D., Meeraus, A. & Suh, J.S., (1981). *Oil Refinery Modeling with the GAMS language*, Tech. rep., Center of Energy Studies, University of Texas.
- Li, J., Karimi, I. & Srinivasan, R., (2010). Recipe Determination and Scheduling of Gasoline Blending Operations. *AIChE Journal*, 56(2), pp.441–465.
- Li, W., Hui, C.-W. & Li, A., (2005). Integrating CDU, FCC and product blending models into refinery planning. *Computers & Chemical Engineering*, 29(9), pp.2010–2028.
- Méndez, C. A., Grossmann, I. E., Harjunkoski, I. & Kabore, P., (2006). A simultaneous optimization approach for off-line blending and scheduling of oil-refinery operations. *Computers and Chemical Engineering*, 30(4), pp.614–634.
- Moro, L., Zanin, A. & Pinto, J., (1998). A planning model for refinery diesel production. *Computers & Chemical Engineering*, 22, pp.1039–1042.
- Pinto, J.M., Joly, M. & Moro, L.F.L., (2000). Planning and scheduling models for refinery operations. *Computers & Chemical Engineering*, 24(9-10), pp.2259–2276.
- Tong, K., Feng, Y. & Rong, G., (2012). Planning under Demand and Yield Uncertainties in an Oil Supply Chain. *Industrial & Engineering Chemistry Research*, 51(2), pp.814–834.