

TAKAGI-SUGENO FUZZY MODELING OF A MAGNETIC LEVITATION SYSTEM

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ABSTRACT

A magnetic levitation system (maglev) is an inherently open-loop unstable system with highly nonlinear dynamics. This property attracts the interest of researchers worldwide in recent years. This paper presents the Takagi-Sugeno (T-S) fuzzy model of the maglev system. The nonlinear dynamical equations of motion of the system were first derived using Lagrangian Approach. Based on the nonlinear dynamical equations, a T-S fuzzy model of the maglev system was then developed. To validate the proposed model, simulations study was carried out and the results show that the T-S fuzzy model of the maglev system can exactly represents the original nonlinear maglev system in the pre-specified domains.

KEYWORDS- Magnetic levitation system, Lagrangian method, mathematical modeling, Takagi-Sugeno fuzzy modeling

INTRODUCTION

Magnetic levitation (maglev) is the use of controlled magnetic fields (or magnetic forces) to cause a magnetic object to float in air in defiance of gravity [1]. In recent years, research on maglev has gained momentum worldwide. This is due to the nonlinear and inherent unstable dynamics of the system, which makes it a perfect test-bed for various control algorithms ranging from conventional theoretical control algorithms to intelligent control algorithms.

Various control strategies have been proposed in literature by numerous researchers to control the maglev system, such as feedback linearization [2, 3, 4], sliding mode [5, 6, 7], PID [8], PD (phase-lead) [9, 10], fuzzy logic [11, 12, 13] and neural network [14, 15]. The control strategies mention above can be divided into linear control and nonlinear control. Linear control methods often linearize the dynamics about a certain operating point. Although this method is sufficient in controlling the system, it is subjected to many restrictions which limits the satisfaction level of the system's performance. In nonlinear control methods the unscathed dynamics model of the system is used in designing the controller.

Although these controllers would provide better performance, the complexity and implementation difficulties of these methods result in most researchers utilizing the linear controllers approach. Linear controllers are simple and easy to implement, but the performance level is limited due to approximations, while nonlinear controllers performed better but are complex and difficult to implement.

Using Takagi-Sugeno fuzzy modeling, it is possible to combine the advantages of both linear and nonlinear controllers (i.e. simplicity

and better performance). The nonlinear model can be linearized about different operating points using T-S fuzzy model to cover the whole spectrum of possibilities so that the linearized model behaves exactly like the original nonlinear model [16], and then the linear control methods can be applied to control the approximated nonlinear system effectively. In T-S fuzzy modeling the local dynamics of each linguistic 'IF THEN' rule originated from human knowledge and experience is represented by a linear system model. The overall fuzzy model of the system is achieved by fuzzy blending of the linear system models [17].

In this paper a nonlinear mathematical model of a laboratory maglev system is derived using Lagrangian approach. The T-S fuzzy model of the maglev system is developed based on the nonlinear dynamical equations. Simulations are then carried out to validate the proposed model.

The rest of the paper is organized as follows: section two gives description of the T-S fuzzy model, section three presents the mathematical modeling of the maglev system, in section four the T-S fuzzy modeling of the maglev system is presented, in section five the simulations result is presented and finally section six gives the conclusion.

TAKAGI-SUGENO FUZZY MODEL

The fuzzy model proposed by Takagi and Sugeno [18] is described by linguistic 'IF THEN' rules originated from human knowledge and experience. The T-S fuzzy model describes the nonlinear system by a set of 'IF THEN' rules with fuzzy sets as antecedents and linear models as the consequents. The overall fuzzy model can then be obtained by fuzzy blending of the linear models. The i^{th} rule of the T-S fuzzy model is of the following form [17, 19]:

$$\text{IF } z_1(t) \text{ is } M_{i1} \text{ AND } \dots \text{ AND } z_n(t) \text{ is } M_{in}, \\ \text{THEN } \dot{x}(t) = A_i x(t) + B_i u(t), \quad i = 1, 2, \dots, q \quad (1)$$

Where M_{in} is fuzzy set, $x(t) \in \mathbf{R}^{n \times 1}$ is the state vector,

$u(t) \in \mathbf{R}^{m \times 1}$ is the input vector, $A_i \in \mathbf{R}^{n \times n}$ is the state matrix and $B_i \in \mathbf{R}^{n \times m}$ is the input matrix, $z_1(t) \dots z_n(t)$ are premise variables that may be function of the state variables, q is the number of rules used. Given a pair of $(x(t), u(t))$, the final outputs of the fuzzy systems are inferred as follows [17, 19]:

$$\&(t) = \frac{\sum_{i=1}^q \omega_i(z(t)) \{A_i x(t) + B_i u(t)\}}{\sum_{i=1}^q \omega_i(z(t))} \quad (2)$$

Where:

$$z(t) = [z_1(t), \dots, z_n(t)] \quad (3)$$

$$\omega_i(z(t)) = \prod_{j=1}^n M_{ij}(z_j(t)) \quad (4)$$

The term $M_{ij}(z_j(t))$ is the grade of the membership function of $z_j(t)$ in M_{ij} .

MATHEMATICAL MODELLING OF MAGLEV SYSTEM

The mathematical model considered in this paper is based on the maglev developed by Sani [20] as shown in figure 1. The free body diagram

of the Maglev System is shown in figure 2. The mathematical model is derived using Lagrange method.



Figure 1: Maglev System developed by Sani [20]

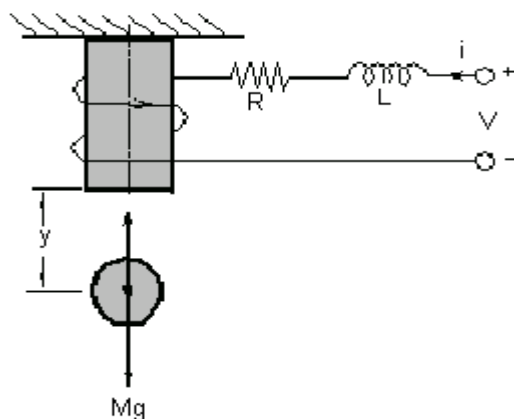


Figure 2: Free body diagram of the Maglev System

The Maglev System can be described based on the following variables and parameters:

M — levitated object mass (kg).

V — voltage (v) applied to the electromagnet coil.

L — coil inductance (H).

R — coil resistance (Ω).

g —gravitational acceleration.

i — current through the coil (A).
 y — levitated object position (m).

The kinetic energy of the system (T) is the sum of the kinetic energy of mass of the levitated object (ball) and that of the inductance of the coil.

$$T = \frac{1}{2} M \dot{y}^2 + \frac{1}{2} L(y) i^2 \quad (1)$$

The potential energy (P) of the system is

$$P = -Mgy \quad (2)$$

The Rayleigh dissipation function is

$$F = \frac{1}{2} Ri^2 \quad (3)$$

The Lagrange function (Γ) is

$$\Gamma = T - P = \frac{1}{2} M \dot{y}^2 + \frac{1}{2} L(y) i^2 + Mgy \quad (4)$$

The inductance $L(y)$ is a nonlinear function of the ball's position y [3], that is

$$L(y) = L_1 + \frac{L_0 y_0}{y} = L_1 + \frac{2C}{y} \quad (5)$$

Where y_0 is an arbitrary reference position for the inductance, L_1 is the coil inductance in the absence of the object, L_0 is additional inductance due to the levitated object and C is the force constant. Substituting (5) in (4) yields

$$\Gamma = \frac{1}{2} M \dot{y}^2 + \frac{1}{2} \left(L_1 + \frac{2C}{y} \right) i^2 + Mgy \quad (6)$$

The Lagrangian equations are given by

$$\frac{d}{dt} \left(\frac{\partial \Gamma}{\partial \dot{y}} \right) - \frac{\partial \Gamma}{\partial y} = 0 \quad (7)$$

$$\frac{d}{dt} \left(\frac{\partial \Gamma}{\partial \dot{i}} \right) + \frac{\partial F}{\partial i} = V \quad (8)$$

The partial derivatives from (6) are,

$$\frac{\partial \Gamma}{\partial \dot{y}} = M \dot{y} \quad (9)$$

$$\frac{\partial \Gamma}{\partial y} = Mg - \frac{C}{y^2} i^2 \quad (10)$$

$$\frac{\partial \Gamma}{\partial i} = \left(L_1 + \frac{2C}{y} \right) i \quad (11)$$

$$\frac{\partial F}{\partial i} = Ri \quad (12)$$

Substituting (9) and (10) in (7) yields

$$M \ddot{y} - Mg + \frac{C}{y^2} i^2 = 0 \quad (13)$$

$$\ddot{y} = g - \frac{C}{M} \left(\frac{i^2}{y^2} \right) = 0 \quad (14)$$

Substituting (11) and (12) in (8) yields

$$\frac{d}{dt} \left(L_1 i + \frac{2C}{y} i \right) + Ri = V \quad (15)$$

$$L_1 \frac{di}{dt} + \frac{2Cy \frac{di}{dt} - 2Ci\dot{x}}{y^2} + Ri = V \quad (16)$$

$$L_1 \frac{di}{dt} + \frac{2C}{y} \frac{di}{dt} - \frac{2Ci\dot{x}}{y^2} + Ri = V \quad (17)$$

$$\left(L_1 + \frac{2C}{y} \right) \frac{di}{dt} - \frac{2Ci\dot{x}}{y^2} + Ri = V \quad (18)$$

It was experimentally found that L_1 is more than 25 times greater than $\frac{L_0 y_0}{y} = \frac{2C}{y}$ [3].

Therefore neglecting $\frac{2C}{y}$, equation (18) becomes

$$L_1 \frac{di}{dt} - \frac{2Ci\dot{x}}{y^2} + Ri = V \quad (19)$$

$$\frac{di}{dt} = \frac{2C}{L_1} \left(\frac{i\dot{x}}{y^2} \right) - \frac{R}{L_1} i + \frac{V}{L_1} \quad (20)$$

Let the states be chosen such that $x_1 = y, x_2 = \dot{x}, x_3 = i$. Thus substituting these in (14) and (20), the state space model of the magnetic levitation system can be written as

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = g - \frac{C}{M} \left(\frac{x_3}{x_1} \right)^2 = 0 \quad (21)$$

$$\dot{x}_3 = \frac{2C}{L_1} \left(\frac{x_2 x_3}{x_1^2} \right) - \frac{R}{L_1} x_3 + \frac{V}{L_1}$$

Using the parameters of table 1 which are based on [20], the numerical model is obtained as:

Table 1: Maglev Parameters

Parameter	Value	Unit
M	0.02312	kg
L_1	0.0425	H
R	3	Ω
g	9.81	m/s^2
C	9.07×10^{-5}	$Nm^2 A^{-2}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{9.81}{x_1} & 0 & -\frac{0.0038x_3}{x_1^2} \\ 0 & \frac{0.0043x_3}{x_1^2} & -70.5882 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 23.5294 \end{bmatrix} V \quad (22)$$

TAKAGI-SUGENO FUZZY MODELING OF MAGLEV

The T-S fuzzy model of the maglev system will be derived from the mathematical model in equations (22). From the dynamical equations (22) the nonlinear terms are $\frac{9.81}{x_1}, \frac{0.0038x_3}{x_1^2}$ and $\frac{0.0043x_3}{x_1^2}$. Therefore, the premise variables are:

$$\begin{aligned} z_1 &= \frac{9.81}{x_1} \\ z_2 &= \frac{0.0038x_3}{x_1^2} \\ z_3 &= \frac{0.0043x_3}{x_1^2} \end{aligned} \quad (23)$$

Substituting (23) in (22) gives:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ z_1 & 0 & -z_2 \\ 0 & z_3 & -70.5882 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 23.5294 \end{bmatrix} v \quad (24)$$

From expert knowledge the current drawn by the electromagnet is chosen to be between 0 and 2A and the displacement is chosen to be between 0.0001m and 0.1m [21], i.e. $x_1 \in [0.0001 \quad 0.1]$ and $x_3 \in [0 \quad 2]$. The maximum and minimum values of nonlinear terms are found to be:

$$z_{1\max} = 98100, \quad z_{2\max} = 0.76 \quad \text{and} \quad z_{3\max} = 0.86$$

$$z_{1\min} = 98.1, \quad z_{2\min} = 0 \quad \text{and} \quad z_{3\min} = 0$$

From the maximum and minimum values, the nonlinear terms can be represented by:

$$z_1(t) = M_{11}(98100) + M_{12}(98.1) \quad (25)$$

$$z_2(t) = M_{21}(0.76) + M_{22}(0) \quad (26)$$

$$z_3(t) = M_{31}(0.86) + M_{32}(0) \quad (27)$$

Where:

$$M_{11} + M_{12} = 1 \quad \Rightarrow \quad M_{11} = 1 - M_{12} \quad (28)$$

$$M_{21} + M_{22} = 1 \quad \Rightarrow \quad M_{21} = 1 - M_{22} \quad (29)$$

$$M_{31} + M_{32} = 1 \quad \Rightarrow \quad M_{31} = 1 - M_{32} \quad (30)$$

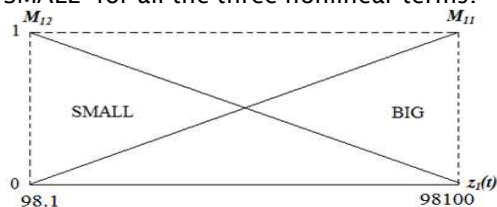
Substituting equations (25)-(27) in (28)-(30) gives the membership functions as:

$$M_{11} = \frac{z_1(t) - 98.1}{98001.9}, \quad M_{12} = \frac{98.1 - z_1(t)}{98001.9}$$

$$M_{21} = \frac{z_2(t)}{0.76}, \quad M_{22} = \frac{0.76 - z_2(t)}{0.76}$$

$$M_{31} = \frac{z_3(t)}{0.86}, \quad M_{32} = \frac{0.86 - z_3(t)}{0.86}$$

Figure 3 illustrates the membership functions. The membership functions are named as "BIG" and "SMALL" for all the three nonlinear terms.



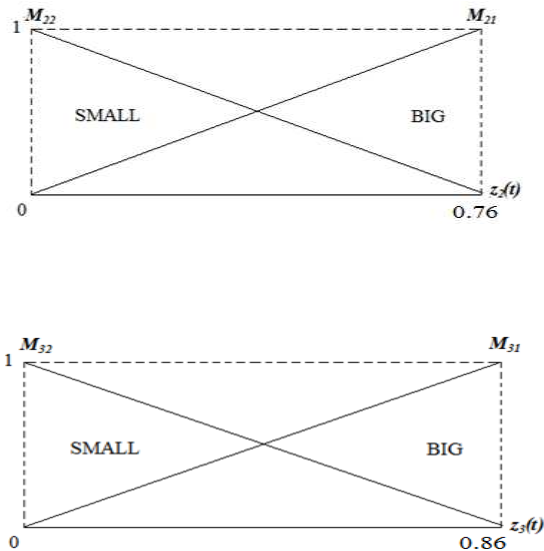


Figure 3: Membership functions

Then the nonlinear model the maglev system is represented by the following T-S fuzzy rules:

- Rule 1:** IF $z_1(t)$ is BIG, $z_2(t)$ is BIG and $z_3(t)$ is BIG,
 THEN $\hat{x} = A_1x + B_1u$
- Rule 2:** IF $z_1(t)$ is BIG, $z_2(t)$ is BIG and $z_3(t)$ is SMALL,
 THEN $\hat{x} = A_2x + B_2u$
- Rule 3:** IF $z_1(t)$ is BIG, $z_2(t)$ is SMALL and $z_3(t)$ is BIG,
 THEN $\hat{x} = A_3x + B_3u$
- Rule 4:** IF $z_1(t)$ is BIG, $z_2(t)$ is SMALL and $z_3(t)$ is
 SMALL, THEN $\hat{x} = A_4x + B_4u$
- Rule 5:** IF $z_1(t)$ is SMALL, $z_2(t)$ is BIG and $z_3(t)$ is BIG,
 THEN $\hat{x} = A_5x + B_5u$
- Rule 6:** IF $z_1(t)$ is SMALL, $z_2(t)$ is BIG and $z_3(t)$ is
 SMALL, THEN $\hat{x} = A_6x + B_6u$
- Rule 7:** IF $z_1(t)$ is SMALL, $z_2(t)$ is SMALL and $z_3(t)$ is
 BIG, THEN $\hat{x} = A_7x + B_7u$
- Rule 8:** IF $z_1(t)$ is SMALL, $z_2(t)$ is SMALL and $z_3(t)$ is
 SMALL, THEN $\hat{x} = A_8x + B_8u$

Where:

$$x(t) = [y \quad \hat{x} \quad i]^T, \quad u(t) = V$$

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 98100 & 0 & -0.76 \\ 0 & 0.86 & -70.5882 \end{bmatrix} \quad A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 98100 & 0 & -0.76 \\ 0 & 0 & -70.5882 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0 & 1 & 0 \\ 98100 & 0 & 0 \\ 0 & 0.86 & -70.5882 \end{bmatrix} \quad A_4 = \begin{bmatrix} 0 & 1 & 0 \\ 98100 & 0 & 0 \\ 0 & 0 & -70.5882 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} 0 & 1 & 0 \\ 98.1 & 0 & -0.76 \\ 0 & 0.86 & -70.5882 \end{bmatrix} \quad A_6 = \begin{bmatrix} 0 & 1 & 0 \\ 98.1 & 0 & -0.76 \\ 0 & 0 & -70.5882 \end{bmatrix}$$

$$A_7 = \begin{bmatrix} 0 & 1 & 0 \\ 98.1 & 0 & 0 \\ 0 & 0.86 & -70.5882 \end{bmatrix} \quad A_8 = \begin{bmatrix} 0 & 1 & 0 \\ 98.1 & 0 & 0 \\ 0 & 0 & -70.5882 \end{bmatrix}$$

$$B_1 = B_2 = B_3 = B_4 = B_5 = B_6 = B_7 = B_8 = [0 \quad 0 \quad 23.5294]^T$$

The T-S fuzzy model of the maglev system is formed by a combination of the eight defined linear models above as follows:

$$\dot{x}(t) = \frac{\sum_{i=1}^8 \omega_i(z(t)) \{A_i x(t) + B_i u(t)\}}{\sum_{i=1}^8 \omega_i(z(t))}$$

Where:

$$\omega_1(z) = M_{11} \times M_{21} \times M_{31}$$

$$\omega_2(z) = M_{11} \times M_{21} \times M_{32}$$

$$\omega_3(z) = M_{11} \times M_{22} \times M_{31}$$

$$\omega_4(z) = M_{11} \times M_{22} \times M_{32}$$

$$\omega_5(z) = M_{12} \times M_{21} \times M_{31}$$

$$\omega_6(z) = M_{12} \times M_{21} \times M_{32}$$

$$\omega_7(z) = M_{12} \times M_{22} \times M_{31}$$

$$\omega_8(z) = M_{12} \times M_{22} \times M_{32}$$

SIMULATION

To validate the proposed T-S fuzzy model of the maglev system, simulations were carried out via Simulink and the results were compared with those from the original nonlinear model of the maglev system. The original nonlinear model of

the maglev system is implemented in Matlab/Simulink based on the nonlinear dynamical equations (22). Based on the information obtained in section IV the fuzzy model of maglev system is implemented in Matlab/Simulink as shown in figure 4 and 5.

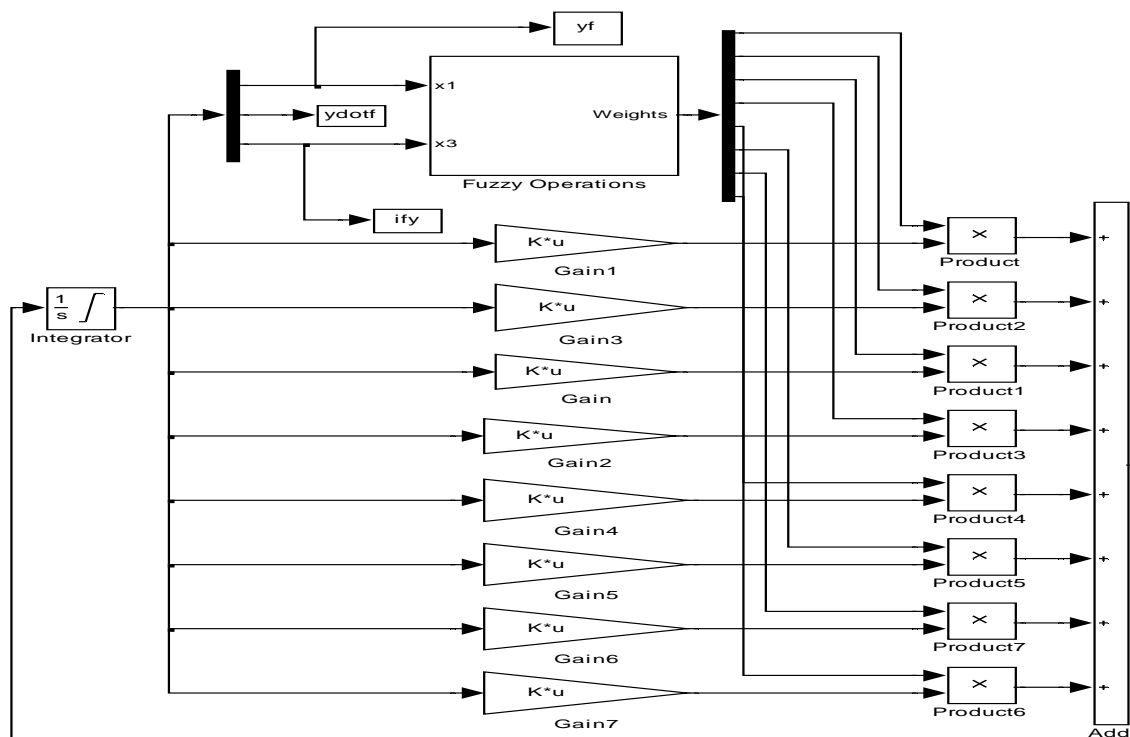


Figure 4: Simulink implementation of the T-S fuzzy model of the maglev system.

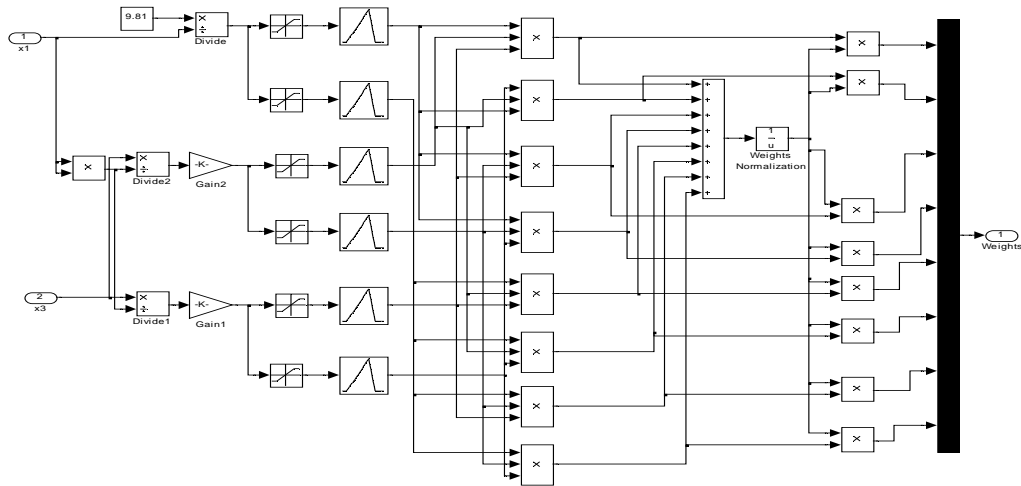


Figure 5: Fuzzy operation block.

The simulation is carried out using the initial condition $x_0 = [0.0001 \ 0 \ 0]^T$. Figure 6 through 8 shows the simulation results. From the simulation results it can be seen that the fuzzy model can exactly represent the nonlinear model within the prescribed region as seen in the ball's position (figure 6), velocity (Figure 7) and current (figure 8).

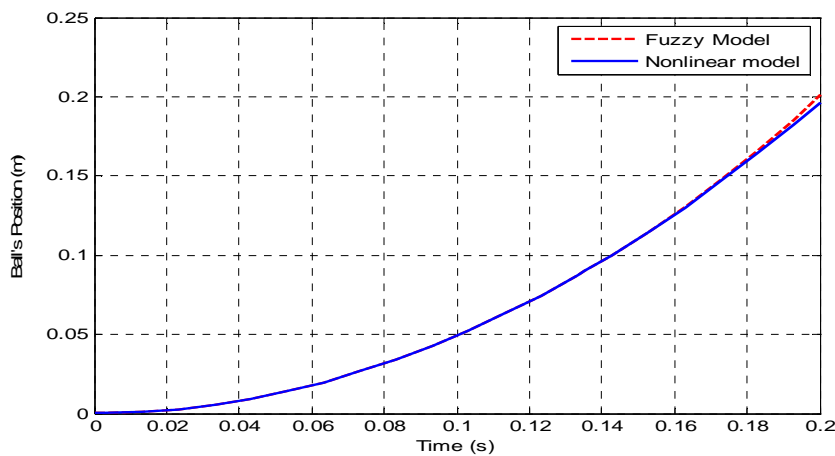


Figure 6: Ball's Position.

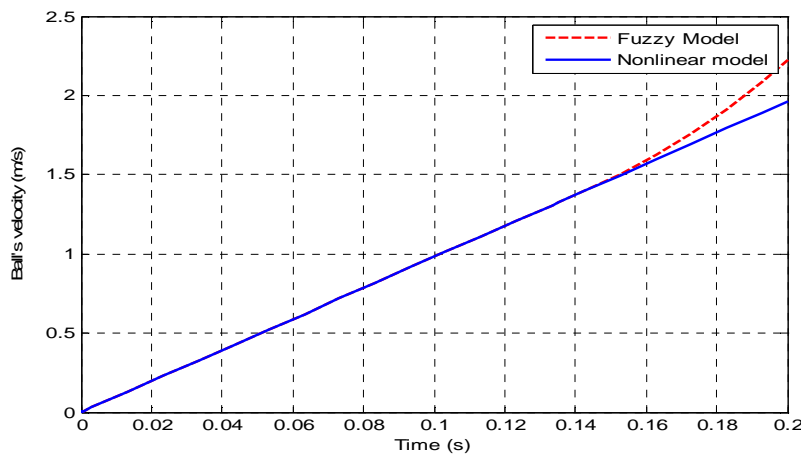


Figure 7: Ball's velocity.

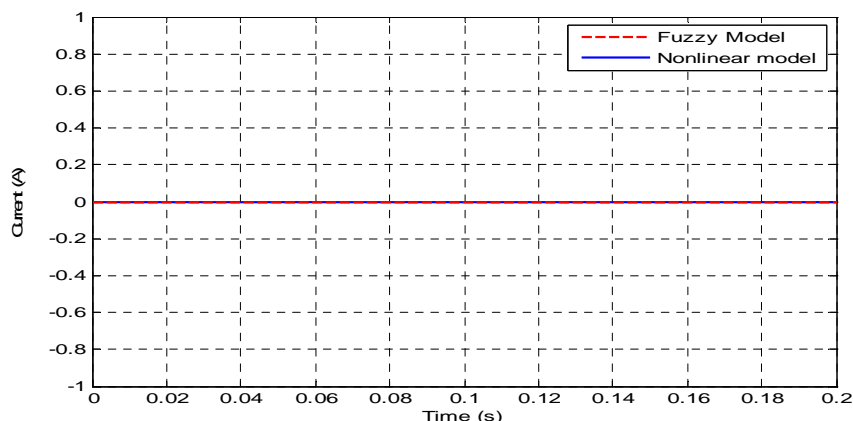


Figure 8: Current

The simulations result discussed above shows that the T-S fuzzy model of the maglev system can represent the nonlinear model of the maglev system to certain degree of accuracy even outside the prescribed region of x_1 and x_2 ;

hence the proposed model is valid to some extent.

For further research, a suitable controller will be design based on the T-S fuzzy model of the maglev system for stabilizing the maglev system.

CONCLUSION

The nonlinear dynamical equations of motion of the maglev system were derived using Lagrange method. Based on the nonlinear dynamical equations, a T-S fuzzy model was developed using 8 fuzzy rules. To validate the proposed model, simulations study was carried out and the

results were compared with those from the original nonlinear model of the maglev. The simulations results show that the T-S fuzzy model of the maglev can exactly represents the original nonlinear maglev system in the pre-specified domains.

REFERENCES

- [1] Richard, C. D.: "Magnetic Levitation", A straightforward and fast information guide to magnetic levitation from ClearlyExplained.com, February 2004.
- [2] I. Ahmad, M. A. Javaid, "Nonlinear Model & Controller Design for Magnetic Levitation System," in *Recent Advances in Signal Processing, Robotics and Automation - 9th WSEAS International Conference on Signal Processing, Robotics and Automation (ISPRA'10)*, Cambridge, 2010, pp. 324-328.
- [3] Barrie, W. and Chiasson J.: "Linear and nonlinear state-space controllers for magnetic levitation". *International Journal of Systems Science*, 1996, 27, (11), pp.1153-1163.
- [4] Joo, S. and Seo J. H.: "Design and Analysis of the Nonlinear Feedback Linearizing Control for an Electromagnetic Suspension System", *IEEE Transactions on Control Systems Technology*, 1997, 5 (1), pp.135-144.
- [5] L. Zhang, Z. Zhang, Z. Long and A. Hao: "Sliding Mode Control with Auto-Tuning Law for Maglev System", *Scientific Research*, 2010, Vol. 2, pp. 107-112.
- [6] Al-Muthairi, N. F. and Zbiri, M. "Sliding mode control of a magnetic levitationsystem", *Mathematical Problems in Engineering*, 2004, Vol. 2, pp.93-107.
- [7] Phuah, J., Lu J. and Yahagi T.: "Chattering Free Sliding Mode Control in Magnetic Levitation System", *IEEJ Transactions on Electronics, Information and Systems*, 2005, Volume 125, Issue 4, pp. 600-606 .
- [8] M. Hypiuseva and J. Osusky "PID Controller Design for Magnetic Levitation Model" *International Conference on Cybernetics and Informatics*, Slovak Republic, Feb. 2010, pp. 1-7.
- [9] M. B. Naumovic and B. R. Veselic: "Magnetic Levitation System in Control Engineering Education", *Automatic Control and Robotics*, 2008, Vol. 7, no. 1, pp. 151-160.
- [10] Wong, T.: "Design of a magnetic levitation system", *IEEE Transaction on Education*, 1986, 29, pp.196-200.
- [11] J.Van Goethem, F.Weber, and G. Henneberger, "Fuzzy control of a magnetic levitation system for a linear drive and comparison with a state control,"in *Proc. Magnetically Levitated Systems and Linear Drives Conf.*, Lausanne, Switzerland, Sept. 2002, PP08103.
- [12] Golob, M.: "Decomposition of a Fuzzy Controller Based on the Inference Break-up Method", in: R. Roy, T. Furuhashi, P.K. Chawdhry (Eds.), *Advances in Soft computing: engineering design and*

- manufacturing, springer, london, 1999, pp. 215-227.
- [13] Tzes, A, Chen, J C, Peng, P Y.: "Supervisory fuzzy control design for a magnetic suspension system". *Proceedings of the 30th IEEE Conference on Fuzzy Systems*, 1994, Vol. 1, pp. 138-143.
- [14] C. Liu, S Zhang, S. Hu and Q wu: "Neural Network Based Robust Nonlinear Control for a Magnetic Levitation System" *International Journal of Innovative Computing, Information and Control*, 2008, vol. 4, no. 9, pp. 2235-2242.
- [15] Hagan, M. Demuth, H. De Jesus, O.: "An Introduction to the Use of Neural Networks in Control Systems", *International Journal of Robust and Nonlinear Control*, 2002, **12**, (11), pp. 959-985.
- [16] R. Carrasco and A. Cipriano: "Fuzzy Logic Based Nonlinear Kalman Filter Applied to Mobile Robots Modelling", *2004 IEEE International Conference on Fuzzy Systems*, 25-29 July, 2004, pp. 1485-1490, Budapest, Hungary.
- [17] K. Mehran "Takagi-Sugeno Modeling for process Control", *Industrial Automatic, Robotics and Artificial Intelligence (EEE8005)*, School of Electrical and Computer Engineering, Newcastle University, 2008.
- [18] T. Takagi and M. Sugeno: "Fuzzy Identification of Systems and its Application to Modeling and Control", *IEEE transactions on Systems, Man, and Cybernetics*, vol. 15 no. 1, pp. 116-132, 1985.
- [19] W. Chang, W. Chang and H. Liu: "Model-Based Fuzzy Modeling and Control for Autonomous Underwater Vehicles in the Horizontal Plane", *Journal of Marine Science Technology*, Vol. 11, No. 3, pp. 155-163, 2003.
- [20] Sani, D. S. "Design and Development of a Magnetic Levitation System" unpublished M.Eng. thesis, Bayero University, Kano, 2004.
- [21] Muhammad, M. "Design of a Neural Network Controller for a Magnetic Levitation System" unpublished M.Eng. thesis, Bayero University, Kano, 2006.